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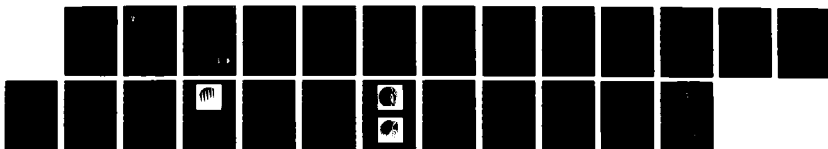
HOLOGRAPHIC PHASE CORRECTION(U) ROYAL SIGNALS AND RADAR
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HOLOGRAPHIC PHASE CORRECTION

Author: C W Slinger

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ROYAL SIGNALS AND RADAR ESTABLISHMENT

Memorandum 4036

TITLE: HOLOGRAPHIC PHASE CORRECTION

AUTHOR: C W Slinger

DATE: June 1987

SUMMARY

The technique of holographic phase correction is described. This scheme offers advantages over more traditional methods, and is applicable to a wide range of problems. The merits and limitations of the technique are investigated by a detailed theoretical analysis, supported by experimental results. Practical problems of implementation are addressed, and it is shown how these limit the accuracy of phase correction.

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HOLOGRAPHIC PHASE CORRECTION

C W Slinger

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1 INTRODUCTION

In many situations it is desirable to convert a wavefront of one form into another - 'wavefront conversion'. The wavefront to be converted, or corrected, may contain known or unknown aberrations. Applications for wavefront correction include aberration compensation for optical imaging⁽¹⁾ and phase correction in coherent optical processing⁽²⁾. In fact, any optical system which processes the complex amplitude of a scene, transparency or transducer may require some form of phase correction or manipulation.

One classical method of phase correction, applicable in a limited set of situations, is the use of a liquid gate - essentially an index matching tank⁽³⁾. This is often used in optical correlation systems⁽⁴⁾. The input is frequently a transparency or liquid crystal display (LCD)⁽⁵⁾. Any phase errors introduced by the transparency or the LCD, due to thickness variations, can largely be removed by immersing them in a tank of index matching liquid. This, however, is frequency bulky and messy, and perfect phase correction requires a high quality input wave and optically flat surfaces for the tank input and output surfaces. Phase variations in the volume of the transducer (due to say, refractive index perturbations) cannot be corrected by the liquid gate.

An alternative method, and the one considered in this publication, is the use of holography as a phase correction technique. This is potentially more powerful, having wide applicability to many systems where phase information is important. The principle will be explained in a qualitative manner in the next section. A more detailed theoretical analysis will then be presented, followed by some experimental results. Implementation will then be discussed and concluding remarks made.

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2 THE PRINCIPLE OF THE HOLOGRAPHIC PHASE CORRECTOR

The holographic principle (eg Ref 6) states that illumination of a hologram by its reference wave will reconstruct the conjugate of the original object wave (Figures 1a and 1b). This holds no matter what form the object or reference waves take. For example, replay of the hologram by its object wave will generate the conjugate of the reference wave. Once this is understood, the principle of the phase corrector can be seen. In this case the reference wave is the wavefront to be corrected, and the object wave is the wavefront of the desired, corrected form (Figure 1c).

As the corrector is holographic, it has several further merits:

- a. It can, in theory, correct any amount of distortion in the wavefront.
- b. It can be compact, low cost and lightweight (the active region of the hologram may be thinner than 5 microns, depending on the recording medium).
- c. The phase correction can be combined with other beam forming functions. This can result in further savings in size and weight for a system. Firstly, the shape of the corrected wavefront need not be planar, the hologram could function as a combined corrector/lens if the original object wave was spherical or cylindrical. Secondly, volume holographic elements are capable of performing several simultaneous beam forming functions, so that, for example, the device could function as a combined beam splitter/multifocus lens/mirror.

Offset against these advantages are several limitations. The device will only function with monochromatic light. It may have efficiency, noise and positional tolerance problems. These latter points will be investigated in more detail in subsequent sections.

An alternative, but equivalent, mode of operation is when the holographic phase corrector is used to pre-aberrate a wavefront before it passes through an aberrating medium. In this case, the hologram is exposed as above, but this time is replayed by the conjugate of the 'phase corrected wave' (terminology as used in Figure 1c). This generates the phase conjugate of the aberrated wave - essentially the wave propagation directions in Figure 1c are reversed. On propagating through the medium, the pre-aberrated wave and the effects of the aberrating medium then cancel to give a phase corrected wave. The hologram is operating in a similar fashion to a phase conjugator, and as such, will not be discussed further here. However, many of the points raised in this Memorandum are directly applicable.

3 THEORY

In this section, an analysis of the phase corrector hologram is made. The aim is to determine whether such a hologram can function as described earlier, and, if it can, to investigate the efficiencies possible and how these efficiencies depend on the recording and material parameters.

The hologram is assumed to be formed between an aberrated 'reference' wavefront (the one to be corrected) and a plane 'object' wave (the one to which the aberrated wavefront is to be corrected). The system is very complex, so several assumptions have to be made to facilitate a tractable solution. The validity of these assumptions are discussed after the derivation of the results. An analytic approach is taken, based on coupled wave theory for 'N wave' holograms⁽⁷⁾. The system is similar to the one analysed in Reference 8.

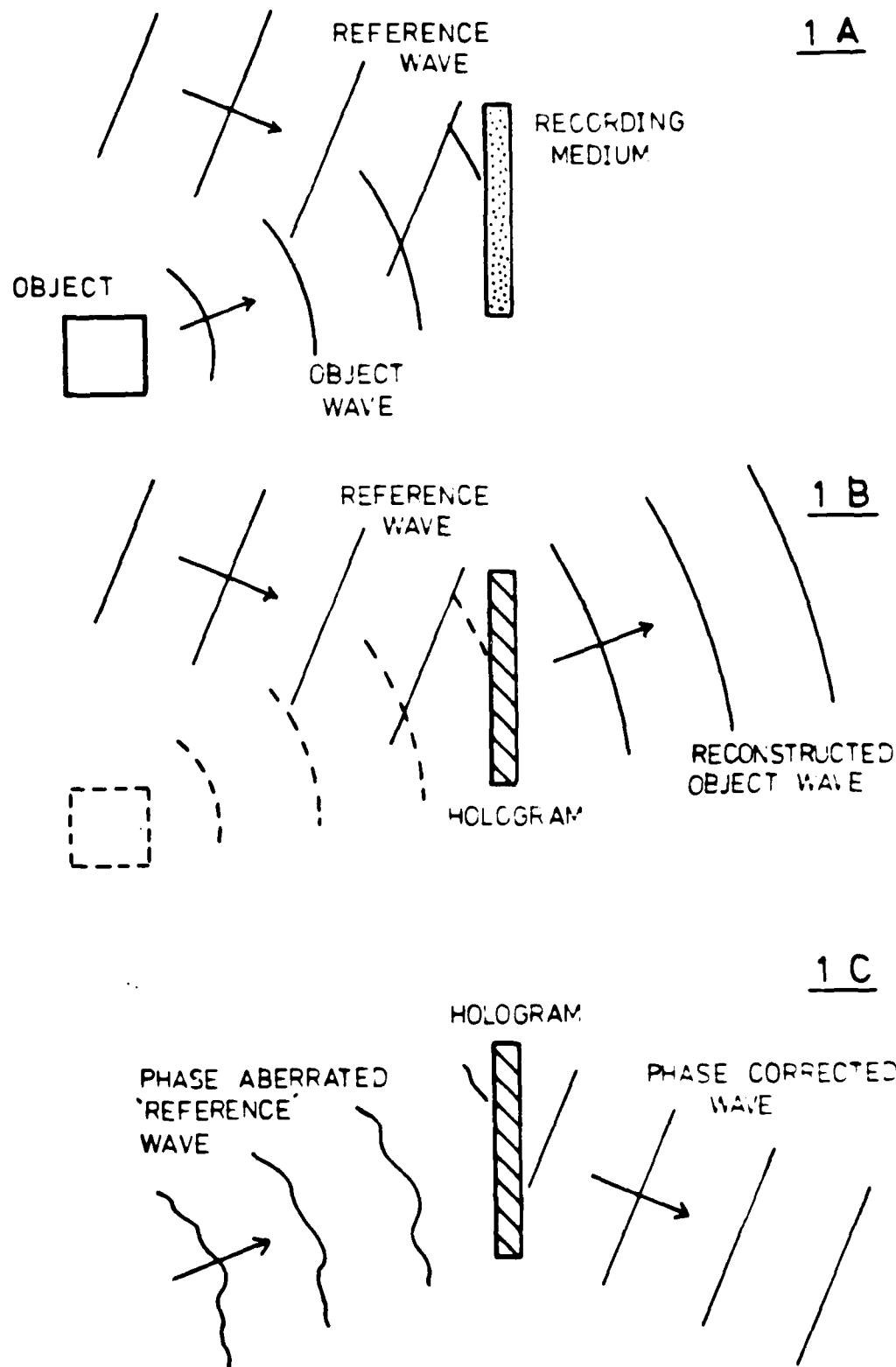


Figure 1. Principle of the phase corrector
 a Recording of a transmission hologram with reference and object waves
 b Replay of the hologram by the reference wave generates the conjugate of the original object wave
 c Replay of a hologram (made using a complicated reference and a simple object wave) by the reference is equivalent to phase correction of the reference wave

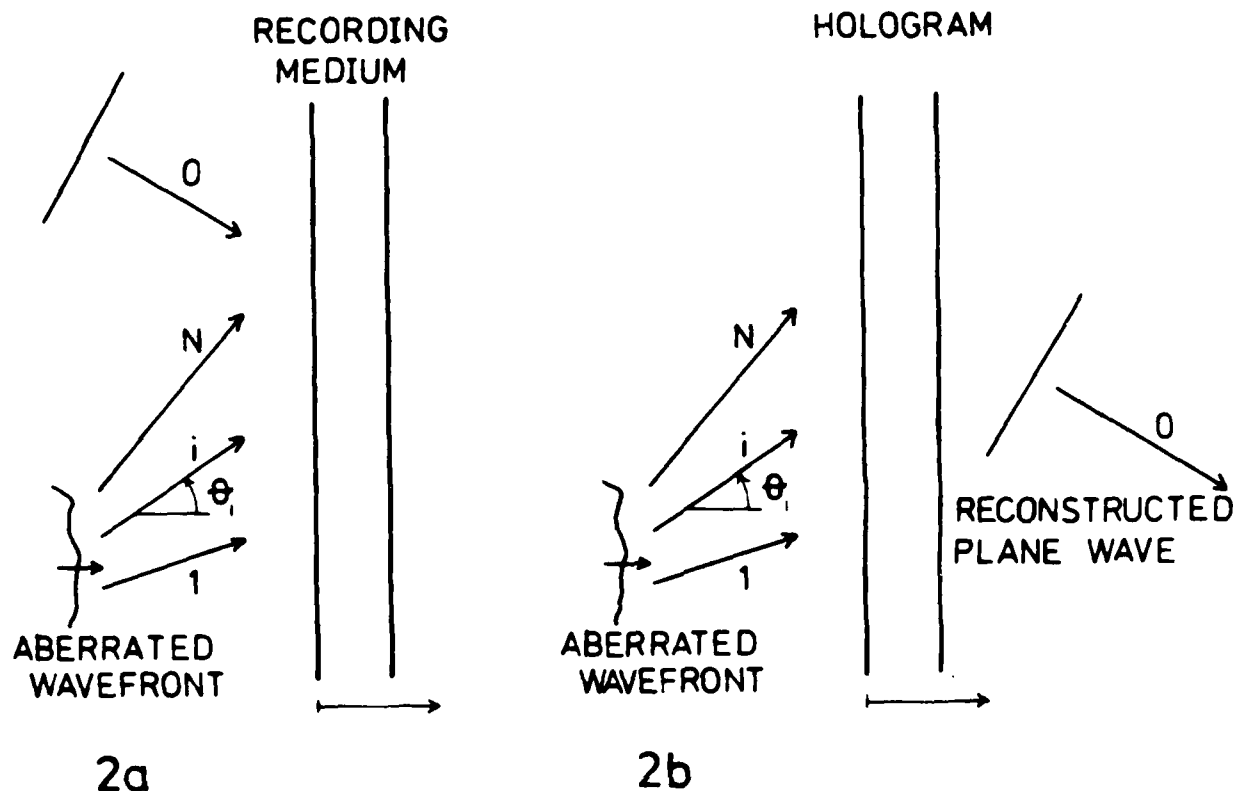


Figure 2. Schematic of the system analysed by the theory
 a Recording of the N wave hologram
 b Replay of the N wave hologram by waves 1, 2, 3, ..., N to reconstruct wave 0

Figure 2a shows a schematic of the recording arrangement. The hologram is recorded with a plane 'object' wave

$$E_0 = A_{00} \exp(-j\beta p_0) \quad (1)$$

and the wavefront to be corrected (the 'reference') is represented by N separate plane waves

$$E_r = \sum_{i=1}^N E_i = \sum_{i=1}^N A_{i0} \exp(-j\beta p_i) \quad (2)$$

where A_{i0} is a complex constant giving the amplitude of the i th wave, $\beta = 2\pi/\epsilon'_0\lambda$, ϵ'_0 is the unperturbed dielectric constant of the recording material and λ is the free space wavelength at recording.

$$p_i = x \cos \theta_i + y \sin \theta_i \quad (3)$$

x and y are rectangular co-ordinates and θ_i is the incident angle of the i th wave in the recording material. All waves are taken to be polarised perpendicular to the plane of recording.

The assumption is then made that the change in dielectric constant produced at recording is proportional to the square of the electric field in the material. Losses due to scatter and absorption at recording are neglected

$$\Delta\epsilon = c \left| \sum_{i=1}^N E_i \right|^2 = c \sum_{i=0}^N \sum_{k=0}^N A_{i0} A_{k0}^* \exp(-j\beta(p_i - p_k)) \quad (4)$$

where c is a constant depending on the exposure and material.

Equation (4) shows that the total number of gratings recorded is $N(N+1)/2$. N of these are the main gratings, produced by interference between wave 0 and each of the N plane waves making up the reference wave. There are also $N(N-1)/2$ intermodulation gratings, formed between each of the N waves of the reference. Intuitively, it could be expected that the main gratings are the ones responsible for the phase correction behaviour of the hologram, in that they will couple power from the N waves of the reference wave to the object wave. The intermodulation gratings, on the other hand, might be expected to add noise and reduce the attainable efficiency of the system. In addition to the main and intermodulation gratings, there are some constant modulation terms ($i=k$). These do not provide any coupling mechanisms between the waves, but may be responsible for some change in the bulk refractive index of the hologram before and after recording.

The developed hologram is now assumed to be illuminated by some subset of the recording waves. Replay is assumed such that the replay waves are on-Bragg with the gratings inside the hologram. Figure 2b illustrates the situation schematically. A solution is assumed in the form

$$E = \sum_{i=0}^N A_i \exp(-j\beta p_i) \quad (5)$$

where A_i is the amplitude of the i th reconstructed wave. Note that by assuming the solution in the form of (5), the existence of higher diffraction orders is neglected, ie volume diffraction is assumed for all the gratings in the system.

Substituting (4) and (5) into the scalar wave equation, neglecting losses due to absorption and scatter, neglecting the second derivatives of A_i (on the grounds that the amplitudes change relatively slowly as the waves propagate through the hologram), and comparing exponentials, leads to the set of equations

$$\frac{dA_i}{dx} + \frac{j}{\cos\theta_i} \sum_{k=0}^N \kappa_{ik} A_k (1 - \delta_{ik}) = 0 \quad (i = 0, 1, 2, \dots, N) \quad (6)$$

where

$$\delta_{ik} = \begin{cases} 0 & k \neq i \\ 1 & k = i \end{cases}$$

and

$$\kappa_{ik} = A_{i0} A_{k0}^* \beta c / 2\epsilon_0$$

is the coupling constant between the i th and k th waves.

The coupled wave⁽⁹⁾ interpretation of this equation is that each wave is coupled to the other waves in the hologram by the relevant coupling constant. As they propagate, there is a continuous interchange of power between them.

In order to find an analytic solution to (6), the assumption is made that each wave (with the exception of wave 0) is coupled to itself in addition to the other waves. Thus

$$\frac{dA_i}{dx} + \left\{ \frac{j}{\cos\theta_i} \sum_{k=1}^N \kappa_{ik} A_k \right\} + \frac{j \kappa_{i0}}{\cos\theta_i} A_0 = 0 \quad (i \neq 0) \quad (7)$$

$$\frac{dA_0}{dx} + \frac{j}{\cos\theta_0} \sum_{k=1}^N \kappa_{0k} A_k = 0 \quad (8)$$

These differential equations are valid for replay of the hologram with any subset of the recording waves, the required solution being determined by the application of the appropriate boundary conditions. For the case of replay by the N waves which make up the wavefront to be corrected by the phase corrector hologram, the boundary conditions are

$$\text{At } x = 0 : \quad A_0 = 0 \quad \text{and} \quad A_i = A_{i0} \quad (i \neq 0) \quad (9)$$

Hence, after some manipulation

$$A_0(x) = -2j \frac{B_0 A_{00}}{u \cos\theta_0} \exp \left\{ -j \frac{K s}{2} x \right\} \sin \left\{ \frac{K u}{2} x \right\} \quad (10)$$

where

$$B_0 = \sum_{k=1}^N A_{k0}^* A_{k0}, \quad K = \frac{\beta c}{2\epsilon_0}, \quad s = \sum_{i=1}^N \frac{|A_{i0}|^2}{\cos\theta_i} \quad (11)$$

and

$$u = \left\{ s^2 + \frac{4s |A_{00}|^2}{\cos^2\theta_0} \right\}^{\frac{1}{2}} \quad (12)$$

Equation (10) gives the amplitude of the corrected wave (equivalent to the reconstructed object wave) with passage through the hologram, as power is coupled to it from the replaying, phase aberrated wave (equivalent to the reference wave). On replay, maximum power occurs in wave 0 when the argument of the sine term in (10) equals $\pi/2$. Beyond this value, overcoupling occurs, and power is coupled out of wave 0 back into the other beams ($i = 1, 2, 3, \dots, N$).

The amplitudes of the waves of the reference beam (ie the phase aberrated wave), after passing through the hologram, can be found to be

$$A_i(x) = \frac{A_{i0}}{s \cos \theta_i} \left\{ (s \cos \theta_i - B_0) + \frac{B_0}{2} \exp \left\{ -j \frac{K s}{2} x \right\} \left\{ u \cos \left[\frac{K u}{2} x \right] - j s \sin \left[\frac{K u}{2} x \right] \right\} \right\} \quad (13)$$

A check on the solutions is provided by power conservation

$$\sum_{i=0}^N |A_i|^2 \cos \theta_i = \text{constant} \quad (14)$$

The solutions (10) and (13) satisfy this condition.

For the case of the phase correction hologram, the efficiency can be defined as the ratio of power in the phase corrected wave (wave 0) to that in the phase aberrated wave prior to correction (A_r). Thus

$$\eta = \frac{|A_0|^2 \cos \theta_0}{\sum_{i=1}^N |A_{i0}|^2 \cos \theta_i} \quad (15)$$

and, substituting (10) and (13) gives

$$\eta = \frac{4r^2 \cos \theta_0}{4r+1} \frac{\left[\sum_{i=1}^N |a_{i0}|^2 \right]^2}{\sum_{i=1}^N |a_{i0}|^2 \cos \theta_i} \sin^2 \left[\frac{K u x}{2} \right] \quad (16)$$

where

$$a_{i0} = \frac{A_{i0}}{A_{00}} \quad \text{and} \quad r = \left[\cos \theta_0 \sum_{i=1}^N \frac{|a_{i0}|^2}{\cos \theta_i} \right]^{-1} \quad (17)$$

Maximum efficiency will occur when

$$K u x = \pi \quad (18)$$

and if θ_i ($i = 1, 2, \dots, N$) are small then (16) simplifies to

$$\eta \approx \frac{4r}{4r+1} \approx \frac{4R}{4R+1} \quad (19)$$

where R is the beam ratio at recording

$$\left\{ |A_{00}|^2 / \sum_{i=1}^N |A_{i0}|^2 \right\}$$

It can be seen immediately that 100% efficiency can only be approached for large beam ratios. This agrees well with the earlier, intuitive, comments concerning the significance of the main and intermodulation gratings. For large beam ratios, the magnitude of the two sets of gratings increase and decrease respectively, thus more power is coupled to wave 0 from waves $i = 1, 2, \dots, N$ rather than these waves redistributing the power amongst themselves. The converse is true for small beam ratios.

It is interesting to note that (19) agrees with the result for the case of replay by wave 0 to reconstruct waves $1, 2, \dots, N$ (7). This gives further confirmation to the virtual equivalence of the phase correction system analysed here and the pre-aberrating, phase conjugate approach.

3.1 VALIDITY OF THE ASSUMPTIONS

Several assumptions were made in the derivation of (16). The main ones will now be discussed in turn:

- a. The material was assumed to be lossless, both at recording and replay and was assumed to have pure phase modulation. In general this is not true, although many phase modulated media come close to this ideal.

Absorption and scatter at recording will result in a decrease in modulation with depth through the recording material. The net result is that maximum efficiency may well occur for $K u x > \pi$.

Losses at replay will, in practice, cause a corresponding decrease in the efficiency predicted by equation (16).

- b. The change in dielectric constant was assumed to be proportional to the square of the electric field at recording (equation (4)). This was equivalent to the assumption that the material has a linear response. In general, this is not true⁽¹⁰⁾. Grating harmonics will result, with the consequence that the higher diffraction orders will be more significant⁽¹¹⁾ (so running counter to other assumptions). Generally, nonlinearities can be reduced by using low exposures and/or pre-exposures, to ensure operation on a relatively linear part of the recording material's transfer characteristic.

- c. The effect of the bulk change in refractive index was neglected. In addition, no mention of recording material thickness change, during development, was made. In many holographic media, there are appreciable changes in either bulk index and/or thickness. This results in it not being possible to replay all the gratings 'on-Bragg' simultaneously with subsequent loss in efficiency. In general, however, these effects can be minimised by suitable choice of recording material or careful attention to processing procedures⁽¹²⁾.

- d. Diffraction orders other than those specified by equation (5) were neglected. At first sight, this might seem to be a sweeping assumption, as there are several diffraction mechanisms by which power can be coupled into orders other than those allowed for in the model. These include diffraction into higher orders⁽¹³⁾ and diffraction by multiple grating interactions^(14,15). In all cases however, these effects are minimised if all the gratings (both the main and the intermodulation gratings) are volume in nature (sometimes referred to as 'optically thick'). Whether a grating is volume or not depends on the

recording and replay geometries and the material characteristics⁽¹⁶⁾. One useful criterion is the Ω parameter, defined as

$$\Omega = K^2 / (2\beta\kappa_1) \quad (20)$$

where K is the modulus of the grating vector ($= 2\pi/L$, where L is the grating period), $\kappa_1 = \beta\epsilon_1' / (4\epsilon_0)$ and ϵ_1' is the amplitude of the phase modulation.

Generally, diffraction in the volume regime occurs if $\Omega > 10$ or so. It is usually a simple matter to arrange that the recording geometry is such that the main gratings are volume. However, this is not so easy for the intermodulation gratings, the reason being that the interbeam angles (ie $|\theta_i - \theta_k|$; $i, k = (1, 2, \dots, N)$) are usually small, hence K_{ik} is also small. Fortunately, as the number of intermodulation gratings rise, then κ_{ik} becomes smaller (due to the finite amount of modulation in the material) thus Ω_{ik} may be sufficiently large to ensure diffraction into a single order. To put this on a more quantitative footing, the amount of power diffracted by the intermodulation gratings into their ± 1 orders will be estimated (as the model allows only for diffraction into their ± 1 orders):

Assuming each of the N waves making up the phase aberrated wave are similar in amplitude, then

$$A_{i0} \propto N^{-1} \quad (i \neq 0) \quad (21)$$

Thus, from equation (6),

$$\kappa_{ik} \propto N^{-2} \quad (i, k \neq 0) \quad (22)$$

If the number of object waves N is large, the modulation κ_{ik} will be small, and thus the diffracted amplitudes will be approximately:

$$A_{ik} \propto \kappa_{ik} \quad (i, k \neq 0) \quad (23)$$

so that the total power diffracted in the ± 1 diffraction orders of the intermodulation gratings I_s will go as

$$I_s \propto \frac{N(N-1)}{2} \kappa_{ik}^2 \approx N^{-2} \quad (24)$$

The power diffracted into these orders therefore goes as N^{-2} and assumption for neglecting these orders is therefore valid for large N .

e. The assumption was made that each of the waves was not only coupled to all the other waves, but was also coupled to itself (with the exception of wave 0). The physical justification for this is that, when N is large, taking one more coupling into account will only have a small effect on the solution. This has been confirmed, for the related case of replay by wave 0 to reconstruct the other N waves (different boundary conditions to the problem here), by numerical solution of equations (6)⁽¹⁷⁾.

The validity of this assumption will increase as N increases. Conversely, for low values of N , the assumption becomes untenable. This can be seen clearly from equations (7) for the limiting case of $N = 1$.

All the difficulties discussed above undermine the analysis to some extent, especially for N small. This is equivalent to saying that the result (equation (16)) is only valid for the case of phase correction of a highly aberrated wavefront. With this in mind, the following example was considered.

3.2 REPLAY EFFICIENCY - AN EXAMPLE

This example represents the phase correction of a highly aberrated wave. Because of the losses inherent in any real system, the results will show the maximum efficiency that is likely to be achievable, and how this will vary with the recording parameters. The range of angles chosen at recording are typical for a practical arrangement.

Consider the situation where wave 0 is incident at angle $\theta_0 = 0$, the N waves making up the wavefront to be corrected are spread (spaced uniformly) over the region $\theta_c \pm \theta_s/2$, where θ_c is the central angle and θ_s is the total angular spread. All the N reference waves are taken to have the same amplitude, $a_{i0} = 1/N$, thus the beam ratio R is unity.

The maximum efficiency η as a function of angular spread θ_s is shown plotted in Figure 3 with θ_c as a parameter. Only a limited range of θ_s is considered, so that the angle θ_i does not exceed 80° and is not less than 5° . Wave 0 and the waves 1, 2, ..., N are then distinguishable and any higher diffraction orders that would arise from the main gratings (if equation (20) becomes such that $\Omega \ll 10$) are kept to a reasonably low level.

It can be seen that, for small angles ($\theta_c = 10^\circ$, $\theta_s < 10^\circ$) the efficiency is near to 80%, as predicted by (19). Higher beam ratios will give higher efficiencies, as the ratio of the strengths of the main gratings to that of the intermodulation gratings increases. For larger angles of θ_c and θ_s , there is a decrease in the available efficiency. For sufficiently large number of waves, it was found that the curves become independent of N . This saturation value depends on θ_c and θ_s , $N=20$ giving an accuracy of better than 1%.

For phase correction of a less aberrated wave, equivalent to small N , hologram efficiencies will approach that given by standard two beam coupled wave theory⁽¹⁸⁾, ie close to 100% may be possible. In this case, the main factors determining the final efficiency are the material losses and the amount of modulation available.

4 EXPERIMENTAL WORK

The aim of the experimental work was to demonstrate that phase correction was possible; to measure the efficiency of the process under various experimental conditions; to compare the results with the theory derived in the previous section and to identify any problems with implementing the phase corrector.

The experimental recording arrangement used is shown schematically in Figure 4a. A diffusing screen was used as a source of the wave to be corrected. The wavefront generated by the diffuser is very complex with a random phase structure. It is probably one of the most difficult phase objects to correct. Initially, a converging wave was used as the object wave (wave 0). This was used in order to demonstrate the ability of the phase corrector to correct a wavefront to one with any desired form.

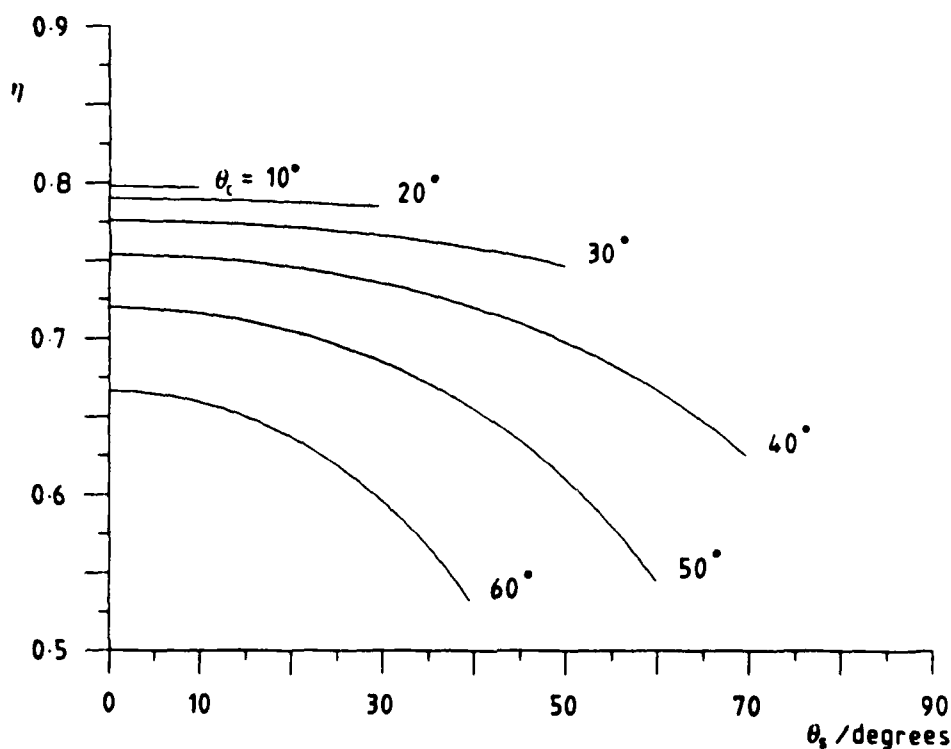


Figure 3. Maximum efficiency of replay η as a function of the angular spread of the object waves as calculated by equation (16), for the system shown in Figure 2. Parameters: $\theta_0 = 0$; $R = 1$.

The recording material used was very high resolution silver halide emulsion (Agfa Gevaert 8E56HD plates). The plates were held in a high precision holder, allowing accurate repositioning after development, and painted black on their rear surfaces prior to exposure, to reduce gratings formed by spurious internal reflections. The recording and replay were made using an argon ion laser, operating single frequency, at 514.5 nm. A range of exposures were used (from 0.1 to 2 mJ cm⁻²), with various beam ratios (R). Standard holographic procedures were followed during recording and processing. Recording parameters were $\theta_0 = -25^\circ$, $\theta_c = 25^\circ$, $\theta_s = 10^\circ$.

4.1 PHASE CORRECTION

After processing, the holograms were replaced in exactly the same position as at recording and replayed with the diffusing screen (Figure 4b). Care was taken to ensure that the diffuser was not moved relative to the plane of the hologram. In all cases, a converging spherical wave was generated by the hologram, ie the wavefront from the diffusing screen was corrected to that of a spherical wave. This successfully demonstrated the principle of the holographic phase corrector. Surrounding the reconstructed wave was a small amount of noise. This was probably caused by

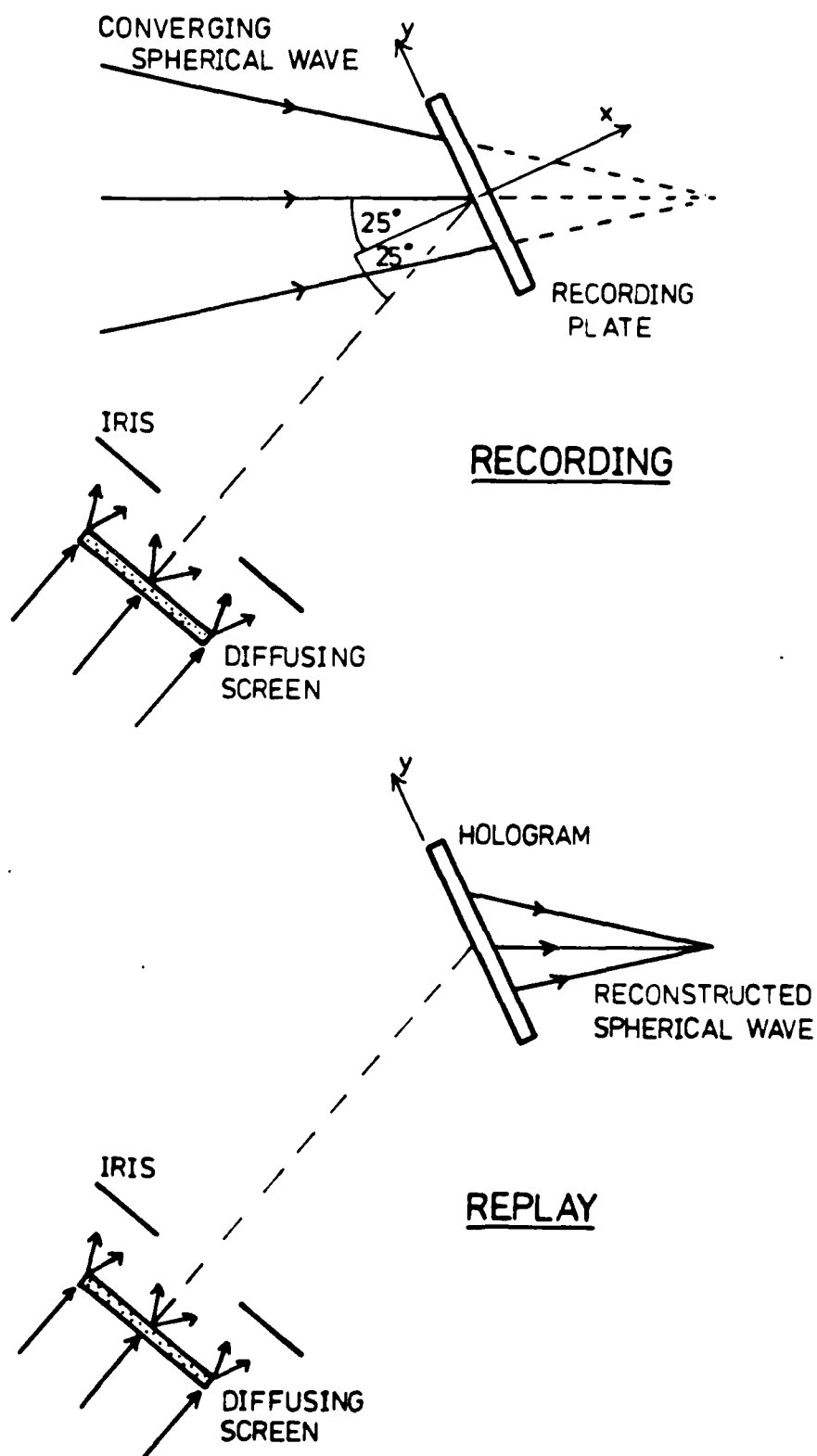


Figure 4. Schematic of the recording and replay arrangements for the phase correction experiments

scatter and multiple grating interactions in the hologram (neglected in the theoretical analysis).

In order to investigate the accuracy of the phase correction, the reconstructed wave was analysed using a simple form of interferometer. Here, holograms were made with a plane wave (instead of the spherical wave) as the object wave, otherwise the recording arrangement was as in Figure 4a. After processing, the hologram was placed in its original recording position and replayed with both the diffuse reference and the plane object wave (Figure 5). The resulting pattern, caused by interference

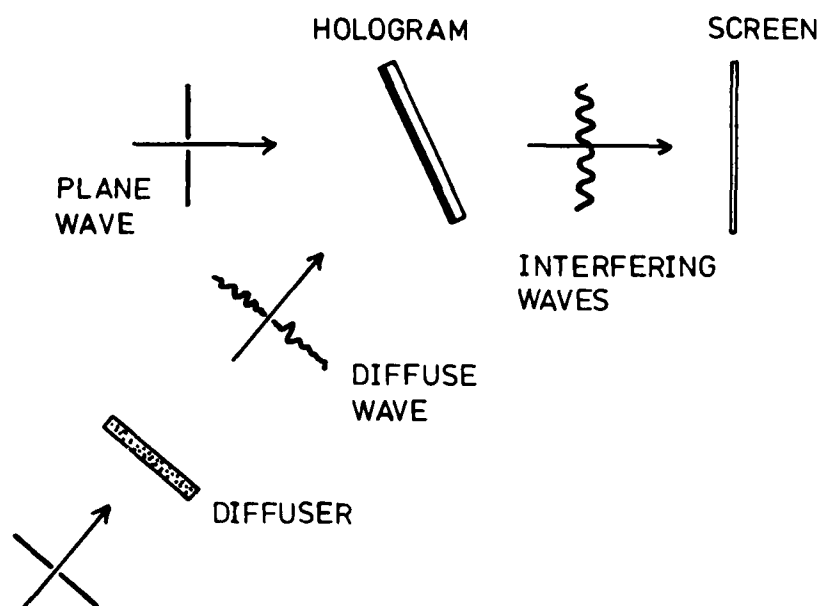


Figure 5. Interferometer arrangement used to investigate the accuracy of the phase correction. The hologram is replayed by both the diffuse wavefront and a plane object wave (ie both the waves used at recording). The resulting interference pattern is imaged on the screen.

between the transmitted and the reconstructed object waves, was then displayed on a screen. Initially, a uniform interference pattern (ie constant intensity) was seen on the screen. No fringes were discernible. This indicated that there was a constant phase difference between the transmitted and reconstructed waves, across the entire active area of the hologram. To illustrate this further, a small tilt was introduced to the plane transmitted wave. A photograph of the resulting fringe system is shown in Figure 6. The fringes are essentially straight and parallel, demonstrating the excellent phase correction achieved by the hologram. It is emphasised that the ability of the hologram to correct the diffuse wavefront is probably the worst case likely to be encountered in practice.



Figure 6. Photograph of a typical interference pattern produced by the system in Figure 5. The parallel fringes indicate good phase agreement between the transmitted object wave and the reconstructed object wave.

4.2 MEASURED EFFICIENCIES

The diffraction efficiencies were measured at several beam ratios. At a beam ratio of 1, the measured efficiencies were in the range of 10% to 13%. For the higher beam ratio of $R = 3$, efficiencies rose to be within the range of 19% to 22%. The noise level in the reconstructed wave was also reduced by the increase in R . The increase in efficiency with beam ratio concurs qualitatively with the predictions of the model, although the measured efficiencies are somewhat lower than those predicted, as expected - for the geometry used in the experiment, equation (16) predicts maximum efficiencies of 80% and 92% for $R = 1$ and 3 respectively.

Some experimentation was made with the processing techniques. The above efficiency figures are for the standard GP61 developer⁽¹⁹⁾ with Agfa-Gevaert G321 fixer and bromine vapour bleach⁽²⁰⁾. This is the usual process for high efficiency transmission holograms in this material. However, using display hologram processing⁽¹²⁾ (CWC2 developer, no fix and parabenzquinone bleach), specially formulated to reduce shrinkage effects, resulted in a general increase in diffraction efficiency. For the $R = 1$ case, the diffraction efficiency was doubled from 10% (transmission developer) to 20% (display hologram developer). This result is in good agreement with previous comments concerning the desirability of replaying with all gratings being on-Bragg simultaneously.

To put these measurements in perspective, the best efficiencies obtained for holographic planar transmission gratings, using the same material and similar processing, are in the region of 58%⁽²¹⁾ (less than 100%, mainly due to scatter, absorption, reflection losses and noise gratings).

4.3 THE REPOSITIONING PROBLEM

Perhaps not surprisingly, the hologram is extremely sensitive to its position relative to the diffuser. Translation by as little as 5 microns or so resulted in the reconstructed wave degenerating progressively into noise. A similar problem has been noted by Collier and Pennington in their work on multicolour holograms using reference beam coding(22). To analyse the degree of repositioning accuracy required, they considered the case of a Fourier Transform Hologram.

The hologram is assumed to be recorded by an object wavefront $f_o(x,y)$ and a reference wavefront $f_r(x,y)$. If the developed hologram is replayed by a new reference wavefront $f'_r(x,y)$, then the output from the hologram is given by

$$[f'_r(x,y) * f_r(x,y)] \otimes f_o(x,y) \quad (25)$$

where $*$ represents correlation and \otimes represents convolution.

If $f'_r(x,y) = f_r(x,y)$, and is such that it has a large spatial bandwidth, then the autocorrelation of $f_r(x,y)$ will be sharply peaked. For example, if the reference's spatial frequency spectrum is flat and of random phase for $-\xi_o/2 < \xi < \xi_o/2$, and zero elsewhere, the autocorrelation function will be of the form $\text{sinc}(\xi_o x)$ and the half-width of the central maximum (ie maximum to first zero) is given by π/ξ_o . If the spatial frequency is high, this halfwidth will be narrow. If $f'_o \neq f_o(x,y)$, the cross correlation will result in a broad random spectrum, and, in general, the output wave will degenerate into noise. The permitted displacement of the hologram (from maximum beam reconstruction to the first zero) will be of the order

$$\pi/\xi_o \approx \lambda/\theta_s \quad (26)$$

where the angular extent of the diffusing screen θ_s is assumed to be small. The above analysis, strictly speaking, applies only to holograms of the Fourier Transform type, but LaMacchia and White found equation (26) to be in good agreement for the case of Fresnel Holograms (the type of hologram used as a phase corrector here), in their work on coded multiple exposure holograms(23).

The tolerance effect was investigated by translating phase corrector holograms made, as before, with the diffusing screen, along the y axis, in both directions. For the case of $\theta_o = -25^\circ$, $\theta_c = 25^\circ$ and $\theta_s = 18^\circ$, it was found that a translation of 5 microns was necessary for the reconstructed converging wave to degenerate into noise. Equation (26) predicts a required translation of about 2 microns. Thus it seems that the Fourier Hologram analysis at least provides an order of magnitude indication of the required repositioning accuracy. Phase correction holograms for less complex wavefronts will have their repositioning tolerances correspondingly relaxed.

5 PRACTICAL CONSIDERATIONS

It has been shown, in earlier sections, that the holographic process is capable of correcting any monochromatic, spatially coherent wavefront into a wavefront of a desired shape. However, there are several practical points to bear in mind when considering the phase correction - in particular, the flatness of the hologram input and output surfaces, and the bulk homogeneity of the holographic medium. These place limits on the ultimate accuracy of the technique.

Consider the case of an aberrated wavefront being corrected to, say, a plane wave. In order to produce the corrected wave on replay, the plane wave and the aberrated wave must interfere in the bulk of the recording medium. Any non-flatness at the input surface of the hologram will result in a non-plane wave interfering with the reference. Subsequent reconstruction of the hologram will not result in a plane wave. Similar effects

can occur at the exit surface of the hologram. In this case the holographic process reconstructs the plane wave, but the exit boundary aberrates it. In addition to the effects of recording material non-flatness, similar considerations apply to any substrate used to support the recording material. Thus the interferometric measurements made in the previous section should, strictly speaking, be regarded as demonstrating the ability of the hologram to correct the diffuse wavefront into a (possibly) aberrated plane wave. Interference of the reconstructed wave and the transmitted wave will still produce uniform fringes, as the plane wave is itself aberrated on transmission through the active region of the hologram and its substrate.

This is demonstrated clearly by the interferometer arrangement illustrated in Figure 7. Here the reconstructed, phase corrected wave, is interfered with a plane wave introduced after the hologram, via a beam splitter. The photograph of the resulting interference pattern is shown in Figure 8. The fringes are curved and non-parallel, indicating the degree of phase error across the reconstructed wave, caused by emulsion and substrate irregularities. An interferogram of a typical float glass substrate is shown in Figure 9, illustrating the degree of optical imperfections that can be expected in practice.

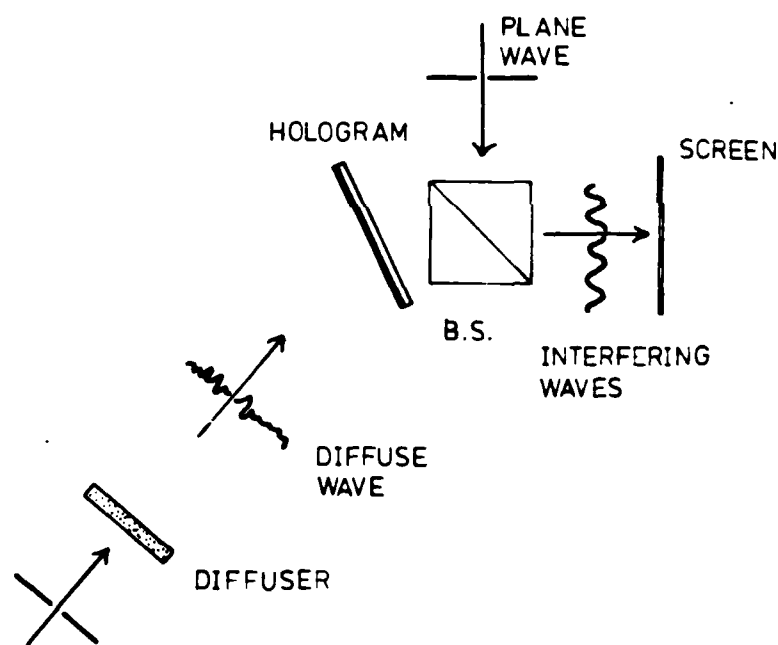


Figure 7. Interferometer arrangement used to assess the absolute accuracy of the phase corrector hologram. It differs from the interferometer depicted in Figure 5 in that the plane wave is introduced after the hologram

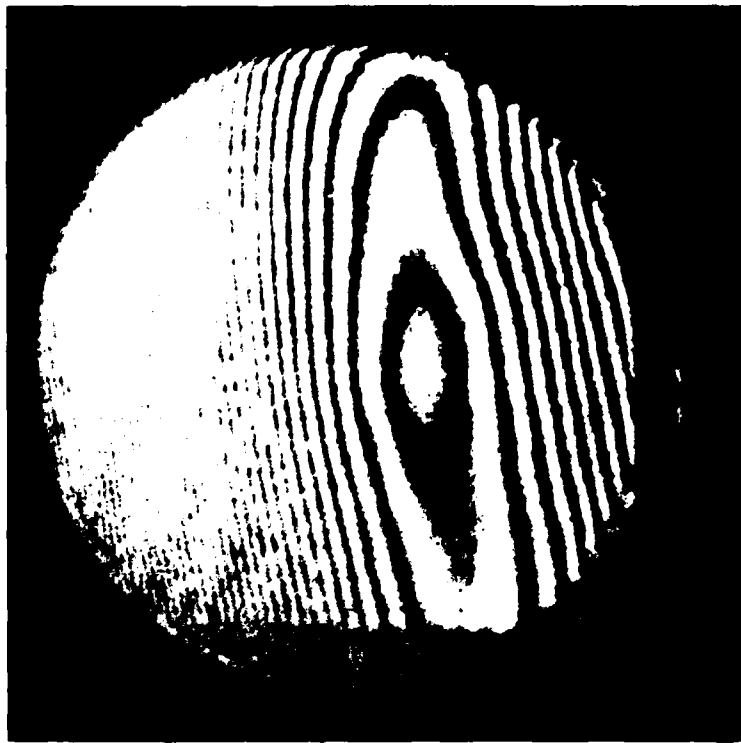


Figure 8. Photograph of a typical interference pattern produced by the system in Figure 7, indicating the phase errors introduced primarily by the hologram substrate.

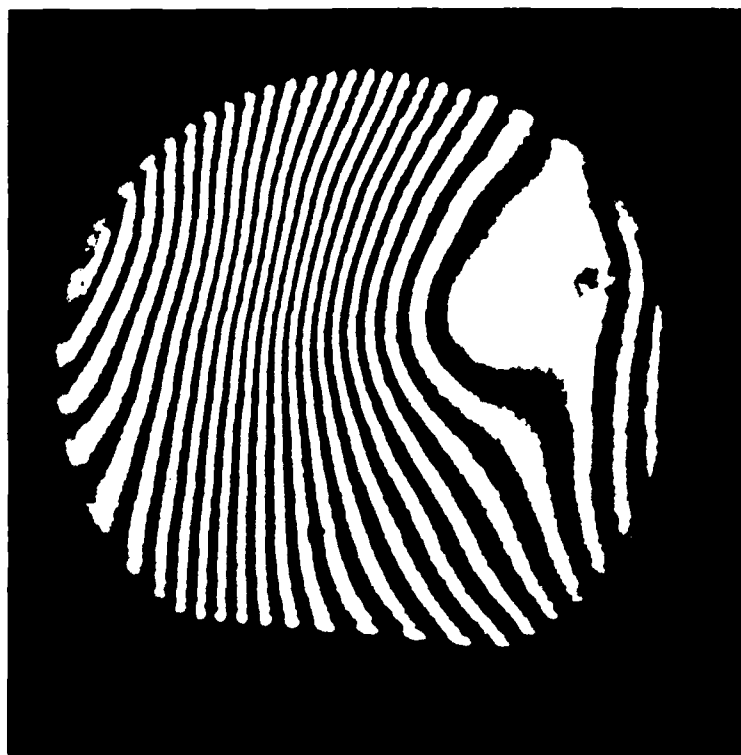


Figure 9. Photograph of fringes of equal thickness from a typical float glass substrate. There is a large degree of thickness variation across the plate.

The problems of substrate and associated imperfections must, however, be looked at in a realistic manner - obviously, the accuracy of the phase correction required depends upon the application. For relatively undemanding situations, no special precautions are necessary. Commercially produced silver halide emulsions on float glass substrates are often adequate⁽²⁴⁾. For more stringent phase correction - say to λ or better, then the flatness requirements may influence the choice of recording material.

Another factor to be considered in a practical phase correction system is the choice of beam ratio R . The theory (equation (19)) and experimental results show that the higher that R is at recording, the higher the efficiency and signal to noise ratio of the corrected wavefront. In practice, the maximum beam ratio that can be used will depend on the amount of modulation available in the recording medium and its thickness - the larger these quantities are, the higher the value of R that can be used. If R is chosen to be too high for the recording medium, saturation will occur, the efficiency will drop and spurious orders will become more significant.

In addition to the above factors, repositioning accuracy will be important. For situations where highly accurate phase correction is required, it may be advisable to develop and process the recording material in situ, without removing it from the system to be corrected. An alternative to this is to use a material that is self developing. Several exist, including some photopolymers and photorefractives.

A particularly promising candidate for high accuracy phase correction is poly(methyl α -cyanoacrylate) (PMCA) doped with parabenzoquinone⁽²⁵⁾. This material is self developing (so there are no repositioning problems), requires no substrate and is capable of being polished to high standards of optical quality. It is currently under investigation for use in a situation where optical phase is used to carry information, and a high degree of phase correction is required.

6 CONCLUDING DISCUSSION

Holography has been shown to be a powerful technique for phase correction. Its ability to correct any wavefront into another, desired, wavefront makes it useful in many systems. The technique has additional advantages of being compact and rugged and is reasonably efficient. Offset against these merits, it is limited to use in systems where monochromatic, spatially coherent light is used. Positional tolerance is also important, and choice of recording material (and substrate, if any) must be made depending on the degree of phase correction required.

Accurate, low noise correction of complex phase objects requires a material of good optical quality and with a high modulation thickness product (so enabling high beam ratios to be used). In many applications, however, commercially available holographic media are adequate. The requirements for high accuracy phase correction (flatness etc) can be regarded as transferring the need for high optical quality from the transducer (where, in many cases, it is impossible or uneconomic) to the hologram.

Finally, it should be noted that the hologram, when formed, corrects only a specific wavefront. This may be regarded as a limitation in that a new hologram has to be recorded each time a different wavefront is to be corrected (unlike the liquid gate, for example). However, if a wavefront φ_1 is corrected to φ_0 , then if the phase of the input wavefront is modified to $\varphi_2 = \varphi_1 + \varphi_\delta$, the corrected wave has this modified phase superimposed, such that it becomes $\varphi_0 + \varphi_\delta$. Thus the hologram can be used to correct the 'quiescent' wavefront error, but at the same time, allows any superimposed phase modulation to be observed. This property may make the phase corrector hologram useful in many situations.

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Abstract <p>The technique of holographic phase correction is described. This scheme offers advantages over more traditional methods, and is applicable to a wide range of problems. The merits and limitations of the technique are investigated by a detailed theoretical analysis, supported by experimental results. Practical problems of implementation are addressed, and it is shown how these limit the accuracy of phase correction.</p>				

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